B.Sc. Part–III Semester–VI Examination

MATHEMATICS

(Graph Theory)

Paper—XII

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory and attempt it once only.

(2) Attempt **ONE** question from each unit.

1. Choose correct alternatives :—

(1) The vertex with degree one is called as :

(a) Even vertex (b) Odd vertex

(c) Pendent vertex (d) Isolated vertex

(2) The maximum number of edges in a simple graph with n vertices is :

(a) $n(n + 1)/2$ (b) $n(n - 1)/2$

(c) $(n + 1)/2$ (d) $(n - 1)/2$

(3) A tree with n vertices has _______ edges.

(a) $n - 1$ (b) $n + 1$

(c) 1 (d) 0

(4) An edge in a spanning tree T is called as :

(a) Branch (b) Chord

(c) Cutset (d) Circuit

(5) The formula $n - e + f = 2$ for planar graph is given by :

(a) Euler (b) Cayley

(c) Kuratowski (d) Hamiltonian

(6) The complete graph of five vertices is called as :

(a) Planar graph (b) Non-planar graph

(c) Vertex graph (d) Bipartite graph

(7) The dimension of the cutset subspace W_s is equal to the _______.

(a) Degree of vertex (b) No. of edges

(c) Rank of the graph (d) Nullity of the graph

(8) Two subspaces W_1 and W_2 are said to be orthogonal to each other iff X.Y = _____. (for all $X \in W_1$, $Y \in W_2$)

(a) 0 (b) 1

(c) X − Y (d) X + Y

(9) In an incidence matrix, a row with all zeros, represent :

(a) Pendent vertex (b) Isolated vertex

(c) Odd vertex (d) Even vertex

(10) In path matrix, each row must contain at least one _______.

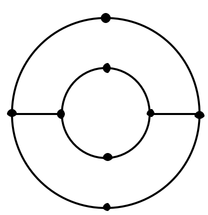
(a) Unit entry (b) Zero entry

(c) 0 (mod 2) entry (d) None of these 10×1=10

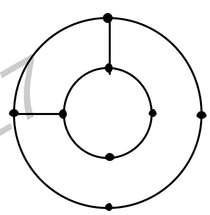
UNIT—I

2. (a) Define (i) Simple graph (ii) Degree of a vertexs. Show that in any graph there are an even number of vertices of odd degree. 2+3

(b) When two graphs are said to be isomorphic ? Whether the following graphs are isomorphic or not ? Explain. 1+4



Graph G_1

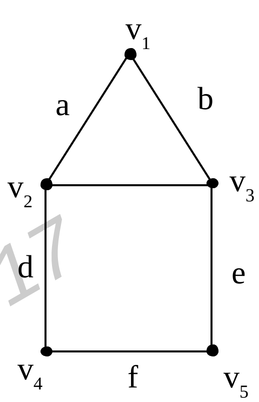


Graph G_2

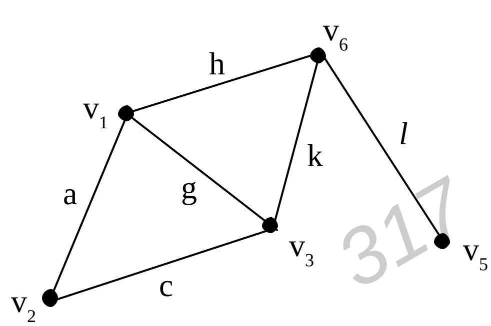
3. (p) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. 5

(q) Define union and intersection of two graphs G_1 and G_2.

From the following figures find (i) $G_1 \cup G_2$ (ii) $G_1 \cap G_2$ (iii) $G_1 \oplus G_2$. 5



Graph G_1



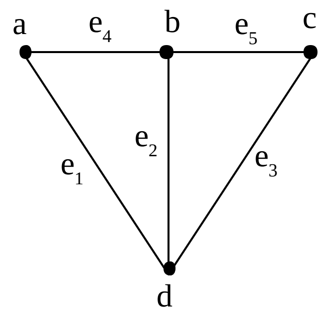
Graph G_2

UNIT—II

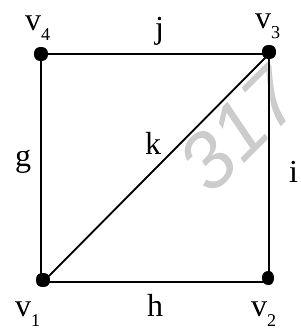
4. (a) Define (i) Binary tree (ii) Rooted tree. Show that there are $\frac{n+1}{2}$ pendent vertices in any binary tree with n vertices. 2+3

   (b) If G is circuit less graph with n vertices and n – 1 edges then prove that there is exactly one path between every pair of vertices in G. 5

5. (p) Sketch all spanning trees of the following graphs :



Graph G_1



Graph G_2

5

   (q) Define centre of a tree and show that every tree has either one or two centres. 1+4

UNIT—III

6. (a) Define planar graph. Prove that complete graph of five vertices is non-planar. 1+4

   (b) If G is a planar connected graph with n vertices, e edges and f faces then prove that n – e + f = 2. 5

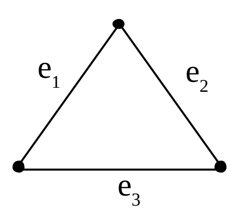
7. (p) Let T_1 and T_2 be two spanning trees of a connected graph G. If edge e is in T_1 but not in T_2 prove that these exists another edge f in T_2 but not in T_1 such that subgraph $(T_1 - e) \cup f$ and $(T_2 - f) \cup e$ are also spanning trees of G. 5

   (q) Define (i) Branch (ii) Chord. Show that every connected graph has at least one spanning tree. 2+3

UNIT—IV

8. (a) Prove that the circuit subspace W_r and the cutset subspace W_s are orthogonal to each other in the vector space of a graph. 5

   (b) For the given graph G, find W_G, W_s, W_r, $W_s \cap W_r$ and $W_s \cup W_r$ with spanning tree $T = \{e_1, e_2\}$. 5

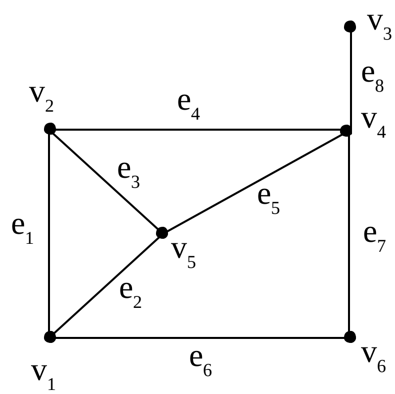


9. (p) Show that the set W_r of all circuit vectors including zero vector in W_G forms a subspace of W_G. 5

(q) Show that subspaces W_r and W_s are orthogonal complements iff $W_r \cap W_s = 0$ i.e. $W_r \cap W_s = \{\phi\}$. 5

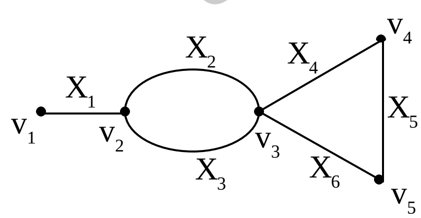
UNIT—V

10. (a) Find Adjacency matrix of the following graph : 5



(b) If A(G) is an incidence matrix of a connected graph G with n vertices then prove that rank of A(G) is n – 1. 5

11. (p) Define circuit matrix. Find the circuit matrix of the following graph : 5



(q) If B is a circuit matrix of a connected graph G with n vertices, e edges then prove that rank of B = e – n + 1. 5